AP Calculus AB
Test: Derivatives
Instructor's Version

Free-Response Question 1 (no calculator)
You may not use your calculator for this question
(When you finish this question, if your test is proctored, ask your proctor for the next section. If you're taking the unproctored version of the test, continue with the next question.)

1. Consider the family of curves described by: $x^2 + xy + ay^2 = b$. ***Remember to justify your answers.***

A. Let $a = 1$ and $b = 7$. Find the two x-intercepts.

Answer: $\left(-\sqrt{7}, 0\right)$ **and** $\left(\sqrt{7}, 0\right)$ (2 points – they do not have to give the answers as ordered pairs.)

$x^2 + x(0) + (0)^2 = 7$ (1.5 points for plugging in values for a, b and y)
$x^2 = 7$ (1.5 points for solving for x)

Note: Award no more than 2 points if no justification is given for answers.

B. Let $a = 1$ and $b = 7$. Using the two points you found in part A, show that the tangents to the curve at these points are parallel, and find the common slope.

Answer: Since $\frac{dy}{dx} = -\frac{2x+y}{2y+x}$ (2 points), the y-coordinates at these points is 0 (1.5 points), the slopes at both points will be -2 (1.5 points).

Note: Award no more than 2 points if no justification is given for answers.

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C. A given curve in the family $x^2 + xy + ay^2 = b$ has a tangent line at point (1, 3) with slope $\frac{-5}{14}$. What are the values of a and b?

Answer: $a = 13/6$ and $b = 47/2$. (5 points)

$$2x + x\frac{dy}{dx} + y + 2ay\frac{dy}{dx} = 0$$

$$\frac{dy}{dx} = \frac{-(2x+y)}{(x+2ay)}$$

$$\frac{-(2+3)}{(1+2a(3))} = \frac{-5}{14}$$

$$a = 13/6$$

$$(1)^2 + (1)(3) + (13/6)(3)^2 = b$$

$$b = 47/2$$

Note: Award no more than 2 points if no justification is given for answers.

Free-Response Question 2 (with calculator)
You may use a calculator on this question.

2. Suppose a coin is dropped from the top of the Empire State building in New York, which is 1,454 feet tall. The position function for free-falling objects is: $s(t) = -16t^2 + v_0 t + s_0$.

A. Determine the position and velocity functions for the coin.

Answer: $s(t) = -16t^2 + 1454$ (1 point)
$v(t) = -32t$ (1 point)

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B. Determine the average velocity of the coin on the interval [1, 3].

Answer: –64 ft/sec (1.5 points for answer, 1 point for units)

average velocity $= \frac{s(3)-s(1)}{3-1} = \frac{1310-1438}{2} = -64$ (1 point for work)

C. Find the instantaneous velocities when $t = 1$ and $t = 3$.

Answer: $v(1) = -32$ ft/s; $v(3) = -96$ ft/s (1.5 points for each answer)

D. At what time is the instantaneous velocity of the coin equal to the average velocity of the coin found in part B?

Answer: When $t = 2$ seconds, $v(2) = -64$ ft/sec. (1 point)

E. What is the name of the theorem that says there must be at least one solution to part D?

Answer: The Mean Value Theorem for Derivatives (1 point)

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F. Find the velocity of the coin just before it hits the ground.

Answer: ≈ -302.051 ft/s (1.5 points for answer, 1 point for sign and units)

$s(t) = -16t^2 + 1454 = 0$ when $t \approx 9.532$ seconds (1 point)
$v(9.5328537982) = -32(9.5328537982) \approx -305.051$ ft/s (1 point for work in evaluating v at time found)

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